Using EDR Data to Calculate Non-EDR Equipped Vehicle Speeds

Sergeant Weston Brown
Scottsdale (Arizona) Police Department
Using EDR Data to Calculate the Speed of a Non-EDR vehicle.

Presented by:
Weston Brown
Scottsdale (Arizona) P.D.
Lots and Lots and Lots of Cars

• As of 2018 model year, over 99% of production vehicles are covered with some type of EDR.
What is $\Delta v$?

What does the $\Delta v$ data mean?

- Let’s first understand what the different types of $\Delta v$ in the EDR mean...
  - Longitudinal Dv – The change in velocity in the longitudinal, or front to back, axis of the vehicle.
What is $\Delta v$?

What does the $\Delta v$ data mean?

- Let’s first understand what the different types of $\Delta v$ in the EDR mean...
  - Lateral Dv – The change in velocity in the lateral, or side to side, axis of the vehicle.
What is $\Delta v$?

What does the $\Delta v$ data mean?
What is $\Delta v$?

What does the $\Delta v$ data mean?

“Right Hand Rule”
SAE J1733
### Sources of Error in $\Delta v$

#### Sources of Error

**$\Delta v$ Error**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Full Scale</th>
<th>Resolution</th>
<th>Accuracy</th>
<th>How Measured</th>
<th>When Updated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta V$</td>
<td>$\pm$ 55.9 mph</td>
<td>0.4 mph</td>
<td>$\pm$ 10%</td>
<td>integrated acceleration</td>
<td>recorded every 10 msec, calculated every 1.25 msec.</td>
</tr>
<tr>
<td>Vehicle speed</td>
<td>158.4 mph</td>
<td>0.6 mph</td>
<td>$\pm$ 4 %</td>
<td>Magnetic pickup</td>
<td>vehicle speed changes by $\geq$ 0.1 mph</td>
</tr>
<tr>
<td>Engine Speed</td>
<td>16383 RPM</td>
<td>1/4 RPM</td>
<td>$\pm$ 1 RPM</td>
<td>Magnetic pickup</td>
<td>RPM changes by $\geq$ 32 RPM.</td>
</tr>
<tr>
<td>Throttle Position</td>
<td>100% Wide open throttle</td>
<td>0.4 %</td>
<td>$\pm$ 5%</td>
<td>Rotary potentiometer</td>
<td>Throttle position changes by $\geq$ 5%.</td>
</tr>
</tbody>
</table>

Sources of Error in $\Delta v$

Sources of Error

$\Delta v$ Error

- This testing showed the average of all $\Delta v$ error to be around 4% and the MSRE to be 11%.

- This testing also found the $\Delta v$ for large but narrow deformation to be unreliable.

<table>
<thead>
<tr>
<th>Test Type</th>
<th>No.</th>
<th>Model</th>
<th>Impact-direction</th>
<th>$\text{Max} \Delta v_{\text{ABS}} \text{ m/s}$</th>
<th>$\text{Max} \Delta v_{\text{REL}} \text{ m/s}$</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frontal</td>
<td>1</td>
<td>O-1 (offset rigid barrier)</td>
<td>front-right</td>
<td>17.4</td>
<td>20.2</td>
<td>-2.8</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>F-1 (concrete block)</td>
<td>front-right</td>
<td>7.3</td>
<td>7.0</td>
<td>-0.3</td>
</tr>
<tr>
<td>Pole</td>
<td>1</td>
<td>P-1 (iron, d=0.3m)</td>
<td>front-center</td>
<td>25.0</td>
<td>17.5</td>
<td>-7.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>P-2 (iron, d=0.3m)</td>
<td>front-right</td>
<td>22.5</td>
<td>20.9</td>
<td>-1.6</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>P-3 (iron, d=0.3m)</td>
<td>side-right</td>
<td>8.0</td>
<td>7.9</td>
<td>-0.1</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>P-4 (concrete, d=0.3m)</td>
<td>front-center</td>
<td>12.6</td>
<td>11.7</td>
<td>-0.9</td>
</tr>
<tr>
<td>Car to car impact</td>
<td>1</td>
<td>A-1</td>
<td>front-left</td>
<td>8.3</td>
<td>8.0</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>A-2</td>
<td>front-right</td>
<td>8.8</td>
<td>7.9</td>
<td>-0.9</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>A-3</td>
<td>front-right</td>
<td>4.5</td>
<td>7.2</td>
<td>-2.7</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>A-4</td>
<td>side-right</td>
<td>3.8</td>
<td>3.5</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>A-5</td>
<td>front-right</td>
<td>16.2</td>
<td>15.9</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>A-6</td>
<td>front-right</td>
<td>15.9</td>
<td>15.6</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>A-7</td>
<td>front-center</td>
<td>12.4</td>
<td>11.0</td>
<td>-1.4</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>A-8</td>
<td>front-center</td>
<td>9.7</td>
<td>8.8</td>
<td>-0.9</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>A-9</td>
<td>front</td>
<td>5.7</td>
<td>5.3</td>
<td>-0.4</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>A-10</td>
<td>front</td>
<td>5.0</td>
<td>5.3</td>
<td>0.3</td>
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<tr>
<td>Multiple rear-end</td>
<td>1</td>
<td>R-1 (1st data)</td>
<td>rear</td>
<td>3.8</td>
<td>4.2</td>
<td>0.4</td>
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<tr>
<td></td>
<td>2</td>
<td>R-1 (2nd data)</td>
<td>rear</td>
<td>6.6</td>
<td>6.9</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>R-2 (1st data)</td>
<td>front</td>
<td>5.7</td>
<td>6.1</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>R-2 (2nd data)</td>
<td>rear</td>
<td>7.5</td>
<td>6.9</td>
<td>-0.6</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>R-3</td>
<td>front</td>
<td>17.7</td>
<td>16.8</td>
<td>-0.9</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>R-4 (1st data)</td>
<td>rear</td>
<td>1.9</td>
<td>1.9</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>R-4 (2nd data)</td>
<td>rear</td>
<td>6.3</td>
<td>6.7</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>R-5 (1st data)</td>
<td>front</td>
<td>4.2</td>
<td>3.2</td>
<td>-10.0</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>R-5 (2nd data)</td>
<td>rear</td>
<td>8.3</td>
<td>9.1</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>R-6</td>
<td>front</td>
<td>16.8</td>
<td>16.0</td>
<td>-0.8</td>
</tr>
</tbody>
</table>

*Average*:

Number of analyzed data:

Root mean square:

*1*: Number of analyzed data

*2*: Vehicle without EDR.

Sources of Error in $\Delta v$

$\Delta v$ Error

- This testing showed the average of all $\Delta v$ error to be around 6% with one $\sigma$ being also 6%.

- Similarly this reference also found the $\Delta v$ for large but narrow deformation to be unreliable.

Sources of Error in $\Delta \nu$

Sources of Error

Over/Under reporting

- How about too much run time in the crash pulse recording algorithm?
- This will cause the $\Delta \nu$ to be over reported.

Note the slope gets very small in this area.

SOURCE: SPD Case #01-XXXXX
Sources of Error in $\Delta v$

**Over/Under reporting**

- Creating a table and graph of your own of the data is a good way to look at the crash pulse if you don’t have acceleration data included.

<table>
<thead>
<tr>
<th>t(s)</th>
<th>S(MPH)</th>
<th>v(fps)</th>
<th>g's</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.01</td>
<td>-1.32</td>
<td>-1.94</td>
<td>-6.01</td>
</tr>
<tr>
<td>0.02</td>
<td>-2.41</td>
<td>-3.53</td>
<td>-4.96</td>
</tr>
<tr>
<td>0.03</td>
<td>-4.39</td>
<td>-6.44</td>
<td>-9.01</td>
</tr>
<tr>
<td>0.04</td>
<td>-7.02</td>
<td>-10.29</td>
<td>-11.97</td>
</tr>
<tr>
<td>0.05</td>
<td>-9.43</td>
<td>-13.82</td>
<td>-10.97</td>
</tr>
<tr>
<td>0.06</td>
<td>-11.63</td>
<td>-17.05</td>
<td>-10.02</td>
</tr>
<tr>
<td>0.07</td>
<td>-11.85</td>
<td>-17.37</td>
<td>-1.00</td>
</tr>
<tr>
<td>0.08</td>
<td>-12.94</td>
<td>-18.97</td>
<td>-4.96</td>
</tr>
<tr>
<td>0.09</td>
<td>-13.38</td>
<td>-19.62</td>
<td>-2.00</td>
</tr>
<tr>
<td>0.1</td>
<td>-14.04</td>
<td>-20.58</td>
<td>-3.00</td>
</tr>
<tr>
<td>0.11</td>
<td>-14.7</td>
<td>-21.55</td>
<td>-3.00</td>
</tr>
<tr>
<td>0.12</td>
<td>-14.48</td>
<td>-21.23</td>
<td>1.00</td>
</tr>
<tr>
<td>0.13</td>
<td>-14.7</td>
<td>-21.55</td>
<td>-1.00</td>
</tr>
<tr>
<td>0.14</td>
<td>-14.7</td>
<td>-21.55</td>
<td>0.00</td>
</tr>
<tr>
<td>0.15</td>
<td>-15.14</td>
<td>-22.20</td>
<td>-2.00</td>
</tr>
<tr>
<td>0.16</td>
<td>-15.36</td>
<td>-22.52</td>
<td>-1.00</td>
</tr>
<tr>
<td>0.17</td>
<td>-15.58</td>
<td>-22.84</td>
<td>-1.00</td>
</tr>
<tr>
<td>0.18</td>
<td>-15.8</td>
<td>-23.16</td>
<td>-1.00</td>
</tr>
<tr>
<td>0.19</td>
<td>-16.02</td>
<td>-23.49</td>
<td>-1.00</td>
</tr>
<tr>
<td>0.2</td>
<td>-16.24</td>
<td>-23.81</td>
<td>-1.00</td>
</tr>
<tr>
<td>0.21</td>
<td>-16.24</td>
<td>-23.81</td>
<td>0.00</td>
</tr>
<tr>
<td>0.22</td>
<td>-16.24</td>
<td>-23.81</td>
<td>0.00</td>
</tr>
<tr>
<td>0.23</td>
<td>-16.24</td>
<td>-23.81</td>
<td>0.00</td>
</tr>
<tr>
<td>0.24</td>
<td>-16.24</td>
<td>-23.81</td>
<td>0.00</td>
</tr>
<tr>
<td>0.25</td>
<td>-16.67</td>
<td>-24.44</td>
<td>-1.96</td>
</tr>
<tr>
<td>0.26</td>
<td>-17.11</td>
<td>-25.08</td>
<td>-2.00</td>
</tr>
<tr>
<td>0.27</td>
<td>-18.65</td>
<td>-27.34</td>
<td>-7.01</td>
</tr>
<tr>
<td>0.28</td>
<td>-19.53</td>
<td>-28.63</td>
<td>-4.01</td>
</tr>
<tr>
<td>0.29</td>
<td>-19.75</td>
<td>-28.95</td>
<td>-1.00</td>
</tr>
<tr>
<td>0.3</td>
<td>-20.18</td>
<td>-29.58</td>
<td>-1.96</td>
</tr>
</tbody>
</table>

Remember:

$$a = \frac{\Delta v}{\Delta t}$$

SOURCE: SPD Case #01-XXXXX
Sources of Error in $\Delta v$

Sources of Error

Over/Under reporting

- Note the two spikes in acceleration data that cannot be seen so easily in the $\Delta v$ only graph supplied by the report.
- In this case a secondary slap is believed to have caused the second spike.
- If this data were relied upon at Maximum $\Delta v$ the impact speed would be over reported significantly.

SOURCE: SPD Case #01-XXXXX
Sources of Error in $\Delta v$

**Sources of Error**

**Over/Under reporting**

- Toyota Gen 1 and 2 have a positive g offset built in.
- Because it is positive it will cause a frontal crash $\Delta v$ to be under reported.
Sources of Error in $\Delta \nu$

Sources of Error

Over/Under reporting

- Toyota Gen 1 and 2 have a positive g offset built in.

- Because it is positive it will cause a frontal crash $\Delta \nu$ to be under reported.
Sources of Error in $\Delta v$

Sources of Error

Clipping/Under reporting

• Mostly seen in really bad, really fast crashes

• Caused when the Acceleration in the crash exceeds the maximum capability of the accelerometer.

• Will always cause the $\Delta v$ to be under-reported.
Sources of Error in $\Delta \nu$

Sources of Error

Truncated $\Delta \nu$/Under reporting

- Most often found in short recording duration EDR’s.
- Caused when the accelerometer stops recording before the crash pulse is over.
- Will always cause the $\Delta \nu$ to be under-reported.

<table>
<thead>
<tr>
<th>Time (milliseconds)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
<th>110</th>
<th>120</th>
<th>130</th>
<th>140</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjusted Algorithm Velocity Change</td>
<td>-1.49</td>
<td>-3.91</td>
<td>-5.88</td>
<td>-9.61</td>
<td>-12.90</td>
<td>-17.51</td>
<td>-23.00</td>
<td>-26.73</td>
<td>-28.70</td>
<td>-31.12</td>
<td>-32.65</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Sources of Error in $\Delta v$

Sources of Error

**Truncated $\Delta v$/Under reporting**

- Most often found in short recording duration EDR’s.
- Caused when the accelerometer stops recording before the crash pulse is over.
- Will always cause the $\Delta v$ to be under-reported.

<table>
<thead>
<tr>
<th>Time (milliseconds)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
<th>110</th>
<th>120</th>
<th>130</th>
<th>140</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjusted Algorithm Velocity Change</td>
<td>-1.49</td>
<td>-3.91</td>
<td>-5.88</td>
<td>-9.61</td>
<td>-12.90</td>
<td>-17.51</td>
<td>-23.00</td>
<td>-26.73</td>
<td>-28.70</td>
<td>-31.12</td>
<td>-32.65</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Sources of Error in $\Delta v$

Sources of Error

Eccentric Collisions/Under reporting

• If the centroid of damage is far from the EDR, this will cause the $Dv$ to be under reported at the EDR.

• This can be adjusted for using the Effective Mass Ratio, EMR.

• More to come on this topic later.
Sources of Error in $\Delta v$

Sources of Error

Restitution

- Restitution, put simply, is the amount of bounce in an object. The closer to 1, or 100%, the more the object bounces back.

Calculating Restitution

$$e = \frac{Separate}{Close}$$

Where $Separate$ is the separation speed and $Close$ is the closing speed.
Sources of Error in $\Delta v$

Restitution

$$e = \frac{\text{Separate}}{\text{Close}}$$

$$e = \frac{7}{70} = 0.1$$

Where *Separation* is the separation speed and *Closing* is the closing speed.
What can we do with this data?

Inline Momentum

Inversely Proportional $\Delta v$

\[ \Delta V_1 = -\Delta V_2 \frac{W_2}{W_1} \]

Inline Closing Speed

Inline Closing Speed = \left[ \frac{1}{1 + e} \right] \left[ |\Delta V_1| + |\Delta V_2| \right]
What can we do with this data?

In-Line Momentum

• Case Study #1
In-Line Momentum EDR
Case Study #1

✓ Chevrolet Cobalt
✓ Chevrolet Trailblazer
✓ Momentum Analysis
  ✓ Need speed of one of the vehicles
    ✓ How
  ✓ Post-Impact velocities
    ✓ Which is most reliable?
✓ Both covered by CDR Tool
  ✓ Precrash data
  ✓ Seatbelt
  ✓ LONGITUDINAL and LATERAL Δ v
Momentum Analysis:

We have the impact speed of the Trailblazer, (0 MPH).

How do we arrive at Post-Impact Speeds?

- Approximate $f$ for the Cobalt
- Yields post impact of 20 MPH for the Cobalt
- Rolling Resistance for the Trailblazer
- Yields Post Impact of 19 MPH for the Trailblazer
- Are these reasonable?

In-Line Momentum Analysis:

- Yields a 51 MPH impact speed for the Cobalt
- Ranging post impact $f$ creates a range of 48 to 53
In-Line Momentum EDR
Case Study #1

✓ What does the CDR data tell us.
  ✓ Seatbelt
  ✓ Pre-crash
  ✓ Longitudinal $\Delta v$ data
    ✓ Collision already colinear
    ✓ Add Post to $\Delta v$
  ✓ Consider Module Error
    ✓ +/- 10%

✓ What do we need?
  ✓ Weights
  ✓ Post Impact $v$'s
  ✓ Cobalt EDR $\Delta v$
  ✓ Longitudinal $\Delta v$ data
  ✓ Consider Module Error
    ✓ +/- 10%
In-Line Momentum EDR
Case Study #1

Cobalt Longitudinal \( \Delta v \) Data

<table>
<thead>
<tr>
<th>Time (milliseconds)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
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<th>100</th>
<th>110</th>
<th>120</th>
<th>130</th>
<th>140</th>
<th>160</th>
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<tbody>
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<td>Longitudinal Axis</td>
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<td>0.00</td>
<td>0.00</td>
<td>-1.36</td>
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<td>-20.34</td>
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<td>-30.50</td>
<td>-33.21</td>
<td>-34.67</td>
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</tr>
<tr>
<td>Recorded Velocity</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time (milliseconds)</td>
<td>160</td>
<td>170</td>
<td>180</td>
<td>190</td>
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<td>260</td>
<td>270</td>
<td>280</td>
<td>290</td>
<td>300</td>
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<tr>
<td>Longitudinal Axis</td>
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<td>-35.25</td>
<td>-35.25</td>
<td>-35.25</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Recorded Velocity</td>
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</tr>
</tbody>
</table>

Longitudinal Crash Pulse

All of the Crash Pulse?
In-Line Momentum EDR
Case Study #1

\[ w_1 = 4810 \]
\[ w_2 = 3228 \]
\[ \Delta v_2 = -35.25 \text{ mph} \]

\[ \Delta V_1 = -\Delta V_2 \frac{W_2}{W_1} \]

\[ \Delta v_1 = -\Delta v_2 \left( \frac{w_2}{w_1} \right) \]

\[ \Delta v_1 = -(35.25) \left( \frac{3228}{4810} \right) \]

\[ \Delta v_1 = (35.25) (0.6711) \]

\[ \Delta v_1 = 23.65 \]

Cobalt Longitudinal \( \Delta v \) Data

<table>
<thead>
<tr>
<th>Time (milliseconds)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
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<th>140</th>
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</thead>
<tbody>
<tr>
<td>Longitudinal Axis Recorded Velocity</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-1.36</td>
<td>-2.71</td>
<td>-5.42</td>
<td>-9.49</td>
<td>-14.23</td>
<td>-20.34</td>
<td>-26.44</td>
<td>-30.60</td>
<td>-33.21</td>
<td>-34.57</td>
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<tr>
<td>Time (milliseconds)</td>
<td>160</td>
<td>170</td>
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<td>270</td>
<td>280</td>
<td>290</td>
<td>300</td>
</tr>
<tr>
<td>Longitudinal Axis Recorded Velocity</td>
<td>-35.25</td>
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<td>-35.25</td>
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</table>
In-Line Momentum EDR
Case Study #1

You must also consider the EDR accuracy of +/- 10%

\[ w_1 = 4810 \]
\[ w_2 = 3228 \]
\[ \Delta v_2 = 35.25 \text{mph} \]

\[ \Delta V_1 = -\Delta V_2 \frac{W_2}{W_1} \]

\[ \Delta v_1 = -\Delta v_2 \left( \frac{w_2}{w_1} \right) \]

\[ \Delta v_1 = -(31.72) \left( \frac{3228}{4810} \right) \]
\[ \Delta v_1 = -(38.77) \left( \frac{3228}{4810} \right) \]

\[ \Delta v_1 = 21.28 \]
\[ \Delta v_1 = 26.02 \]

Calculated \( \Delta v \) range for Trailblazer

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
<th>Target</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta v )</td>
<td>21.82</td>
<td>23.65</td>
<td>26.02</td>
</tr>
</tbody>
</table>
In-Line Momentum EDR
Case Study #1

✓ $\Delta v$ of each vehicle
✓ A restitution coefficient, (we will use 10%)

Inline Closing Speed

$$v_{close} = \left[ \frac{1}{1+e} \right] |\Delta v_1| + |\Delta v_2|$$

Target $\Delta v$

$$v_{close} = \left[ \frac{1}{1+e} \right] [23.65] + [-35.25]$$

$$v_{close} = [0.909] [58.9]$$

$$v_{close} = 53.54 \text{ mph}$$
In-Line Momentum EDR
Case Study #1

✓ ∆\(v\) of each vehicle
✓ A restitution coefficient, (we will use 10%)

Inline Closing Speed = \[\frac{1}{1+e}\] \(\left[|\Delta V_1| + |\Delta V_2|\right]\)

Low ∆\(v\)
\[v_{\text{close}} = \frac{1}{1+e} \left[|\Delta v_1| + |\Delta v_2|\right]\]
\[v_{\text{close}} = \frac{1}{1+0.1} \left[21.82 + (-35.25)\right]\]
\[v_{\text{close}} = [0.909][57.07]\]
\[v_{\text{close}} = 51.87 \text{ mph}\]

High ∆\(v\)
\[v_{\text{close}} = \frac{1}{1+e} \left[|\Delta v_1| + |\Delta v_2|\right]\]
\[v_{\text{close}} = \frac{1}{1+0.1} \left[26.02 + (-35.25)\right]\]
\[v_{\text{close}} = [0.909][61.27]\]
\[v_{\text{close}} = 55.69 \text{ mph}\]

Calculated closing speed range

<table>
<thead>
<tr>
<th>Low</th>
<th>Target</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>51.87</td>
<td>53.54</td>
<td>55.69</td>
</tr>
</tbody>
</table>
In-Line Momentum EDR
Case Study #2

✓ Delta-v Analysis
✓ Pre-crash Data Analysis

EDR Analysis

- Last Data Point: 53  53
- +/- ABS: N/A  N/A
- Braking: -7.5  -0
- Speedo Error: -2.1  +2.1
- 43 to 55

Brake Switch indicates off throughout recording.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>-5 sec</th>
<th>-4 sec</th>
<th>-3 sec</th>
<th>-2 sec</th>
<th>-1 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle Speed (MPH)</td>
<td>53</td>
<td>53</td>
<td>53</td>
<td>53</td>
<td>53</td>
</tr>
<tr>
<td>Engine Speed (RPM)</td>
<td>1866</td>
<td>1792</td>
<td>1792</td>
<td>1792</td>
<td>1792</td>
</tr>
<tr>
<td>Percent Time in Pedal</td>
<td>33</td>
<td>32</td>
<td>33</td>
<td>33</td>
<td>32</td>
</tr>
<tr>
<td>Accelerator Pedal Position (percent)</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Anti-lock Brake System Active (If Equipped)</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Lateral Acceleration (gels/hr/If Equipped)</td>
<td>Invalid</td>
<td>Invalid</td>
<td>Invalid</td>
<td>Invalid</td>
<td>Invalid</td>
</tr>
<tr>
<td>Yaw Rate (degrees per second) (If Equipped)</td>
<td>Invalid</td>
<td>Invalid</td>
<td>Invalid</td>
<td>Invalid</td>
<td>Invalid</td>
</tr>
<tr>
<td>Steering Wheel Angle (degrees) (If Equipped)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
In-Line Momentum EDR
Case Study #1
Summary

<table>
<thead>
<tr>
<th>Momentum Analysis</th>
<th>$\Delta v$ Derived</th>
<th>EDR Pre-crash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min: 48 MPH</td>
<td>Min: 51 MPH</td>
<td>Min: 43 MPH</td>
</tr>
<tr>
<td>Max: 53 MPH</td>
<td>Max: 56 MPH</td>
<td>Max: 55 MPH</td>
</tr>
</tbody>
</table>

✔ What is the Speed range you should use?
  ✔ Overall range 43-55
  ✔ 51 – 53 is compelling to the lay person.
  ✔ The Speeds most closely overlap in this range.
In-Line Momentum

Case Study #2
In-Line Momentum EDR
Case Study #2

1996 Chevrolet Tahoe
Nissan Pathfinder
Momentum Analysis
  Need speed of one of the vehicles
  How
  Post-Impact velocities
  Which is most reliable?
Tahoe covered by CDR Tool
  Seatbelt
  Ignition Cycles at Deployment
  Ignition Cycles at Investigation
  LONGITUDINAL Δ v
No Precrash information
  Is this useful
  If so, how do I use it.
In-Line Momentum EDR
Case Study #2

✓ Momentum Analysis:
  ✓ How do we arrive at a speed for one of the vehicles?
    ✓ Impact speed of vehicle #2
      ✓ Acceleration from stop sign, approximated 0.15
      ✓ Yields an impact speed of 25 MPH
  ✓ How do we arrive at Post-Impact Speeds?
    ✓ Spin analysis on the Pathfinder
    ✓ Yields a post impact speed of 34 MPH
    ✓ Is this reasonable for the Tahoe?
    ✓ Assign to the Tahoe
  ✓ In-Line Momentum Analysis:
    ✓ Police Tahoe travelling at 39 MPH.
In-Line Momentum EDR
Case Study #2

✓ Momentum Analysis:
  ✓ Range the impact speeds using reasonable f values.
    ✓ 0.1 to 0.25 acceleration factor
    ✓ Yields impact velocity of 20.4 to 32.2 mph respectively
  ✓ Impact velocity effect.
    ✓ 42MPH for 20.4 Impact
    ✓ 22 MPH for 32.2 MPH Impact
✓ In-Line Momentum Analysis:
  ✓ Does any of this make sense?
## In-Line Momentum EDR

### Case Study #2

**All of the Crash Pulse?**

### Tahoe Longitudinal $\Delta v$ Data

<table>
<thead>
<tr>
<th>Time (milliseconds)</th>
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<th>30</th>
<th>40</th>
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<tbody>
<tr>
<td>Recorded Velocity Change (MPH)</td>
<td>-0.66</td>
<td>-2.65</td>
<td>-5.05</td>
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<td>-9.43</td>
<td>-10.75</td>
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### Tahoe Crash Pulse

![Tahoe Crash Pulse Graph](image)

- Blue line: $S$(MPH)
- Red line: $g$'s
In-Line Momentum EDR
Case Study #2

✓ What do we adjust for?
  ✓ EDR error (+/- 10%)
  ✓ Restitution
✓ Yields
  ✓ Low $\Delta v$
  ✓ Target $\Delta v$
  ✓ High $\Delta v$

Tahoe $\Delta v$ range:

$\Delta v_{1Low} = -14.71\text{mph}$

$\Delta v_{1Tgt} = -16.35\text{mph}$

$\Delta v_{1High} = -17.43\text{mph}$

Tahoe Longitudinal $\Delta v$ Data

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In-Line Momentum EDR

Case Study #2

\[ w_1 = 3980 \]
\[ w_2 = 6080 \]
\[ \Delta v_{1_{Low}} = -14.71\text{mph} \]
\[ \Delta v_{1_{Tgt}} = -16.35\text{mph} \]
\[ \Delta v_{1_{High}} = -17.43\text{mph} \]

Calculate Pathfinder \( \Delta v \)

\[ \Delta v_{1_{Tgt}} = -\Delta v_2 \left( \frac{w_2}{w_1} \right) \]
\[ \Delta v_{1_{Tgt}} = -(-16.35) \left( \frac{6080}{3980} \right) \]
\[ \Delta v_{1_{Tgt}} = (16.35)(1.52) \]
\[ \Delta v_{1_{Tgt}} = 24.97\text{mph} \]

Tahoe Longitudinal \( \Delta v \) Data

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<tbody>
<tr>
<td>Recorded Velocity Change (MPH)</td>
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In-Line Momentum EDR

Case Study #2

\[ w_1 = 3980 \]
\[ w_2 = 6080 \]

Tahoe \( \Delta v \) range:
\[ \Delta v_{1,\text{Low}} = -14.71 \text{mph} \]
\[ \Delta v_{1,\text{Tgt}} = -16.35 \text{mph} \]
\[ \Delta v_{1,\text{High}} = -17.43 \text{mph} \]

Pathfinder \( \Delta v \) range:
\[ \Delta v_{2,\text{Low}} = 22.47 \text{mph} \]
\[ \Delta v_{2,\text{Tgt}} = 24.97 \text{mph} \]
\[ \Delta v_{2,\text{High}} = 26.62 \text{mph} \]

Tahoe Longitudinal \( \Delta v \) Data

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In-Line Momentum EDR

Case Study #2

\[ w_1 = 3980 \]
\[ w_2 = 6080 \]

Tahoe \( \Delta v \) range:
\[ \Delta v_{1Low} = -14.71 \text{mph} \]
\[ \Delta v_{1Tgt} = -16.35 \text{mph} \]
\[ \Delta v_{1High} = -17.43 \text{mph} \]

Pathfinder \( \Delta v \) range:
\[ \Delta v_{2Low} = 22.47 \text{mph} \]
\[ \Delta v_{2Tgt} = 24.97 \text{mph} \]
\[ \Delta v_{2High} = 26.62 \text{mph} \]

Tahoe Longitudinal \( \Delta v \) Data

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| Time (milliseconds) | 160 | 170 | 180 | 190 | 200 | 210 | 220 | 230 | 240 | 250 | 260 | 270 | 280 | 290 | 300 |
|--------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
In-Line Momentum EDR
Case Study #2

\[ v_f - v_0 = \Delta v \]
\[ v_f - v_0 = \Delta v \]
\[ v_3 - v_1 = \Delta v_{lx} \]
In-Line Momentum EDR
Case Study #2

\[ v_3 - v_1 = \Delta v_{1x} \]

Solve for \( v_1 \)

\[ v_1 = v_3 - \Delta v_{1x} \]
In-Line Momentum EDR
Case Study #2

Pathfinder $\Delta v$ range:
$\Delta v_{2,\text{Low}} = 22.47 \text{ mph}$
$\Delta v_{2,Tgt} = 24.97 \text{ mph}$
$\Delta v_{2,\text{High}} = 26.62 \text{ mph}$

Pathfinder impact speed:
$v_{2,Tgt} = 9.23 \text{ mph}$
$v_{2,\text{Low}} = 7.58 \text{ mph}$
$v_{2,\text{High}} = 11.73 \text{ mph}$

Tahoe Longitudinal $\Delta v$ Data

<table>
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<th>40</th>
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|---------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
In-Line Momentum EDR
Case Study #2

Tahoe $\Delta v$ range:
\[ \Delta v_{\text{Low}} = -14.71 \text{mph} \]
\[ \Delta v_{\text{Tgt}} = -16.35 \text{mph} \]
\[ \Delta v_{\text{High}} = -17.43 \text{mph} \]

Tahoe impact speed:
\[ v_{\text{Low}} = 48.91 \text{mph} \]
\[ v_{\text{Tgt}} = 50.55 \text{mph} \]
\[ v_{\text{High}} = 51.63 \text{mph} \]
**In-Line Momentum EDR**  
**Case Study #2**

**Summary**

<table>
<thead>
<tr>
<th>Momentum Analysis</th>
<th>EDR Derived</th>
<th>✓ Which do you believe and why?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min: 22 MPH</td>
<td>Min: 48 MPH</td>
<td>✓ Momentum</td>
</tr>
<tr>
<td>Max: 42 MPH</td>
<td>Max: 51 MPH</td>
<td>✓ Calculated impact velocity of 20-32 MPH for Pathfinder</td>
</tr>
<tr>
<td></td>
<td></td>
<td>✓ $\Delta v$ derived.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>✓ Utilize the Post impact combined with delta v to obtain a speed for the Pathfinder and the Tahoe.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>✓ Yields impact velocity of 11-15 MPH for Pathfinder</td>
</tr>
<tr>
<td></td>
<td></td>
<td>✓ Yields impact velocity of 48-51 MPH for the Tahoe</td>
</tr>
</tbody>
</table>

**Momentum EDR Derived**

21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52
In-Line Momentum EDR
Case Study #2
Non-collinear Collisions

Eccentric Collisions/Under reporting

• If the centroid of damage is far from the EDR, this will cause the $\Delta v$ to be under reported at the EDR.

• This can be adjusted for using the Effective Mass Ratio, EMR.
Non-collinear Collisions

Effective Mass Ratio ($\lambda$)

- An adjustment for the $\Delta v$ is necessary when the PDOF does not go through the vehicles COM.
Non-collinear Collisions

Calculating Effective Mass Ratio

• Two ways
  – Calculate mathemagically
  – Estimate (Rich Estimation)
Non-collinear Collisions

Calculating Effective Mass Ratio

- Calculate Mathemagically
  - $k$ is radius of gyration
  - $h$ is the Moment arm of the crash impulse
- How do you find $k^2$?

$$\gamma = \frac{k^2}{k^2 + h^2}$$
Non-collinear Collisions

Calculating Effective Mass Ratio

• What is the *Radius of Gyration*?
  - A length that represents the distance in a rotating system between the point about which it is rotating and the point to or from which a transfer of energy has the maximum effect

• How do you find $k^2$?

$$k^2 = \frac{I_y g}{W}$$
Non-collinear Collisions

Calculating Effective Mass Ratio

• Calculate radius of gyration
  – $I_\gamma$ is the yaw moment of inertia
  – $g$ is gravity
  – $w$ is the weight of the vehicle

$$k^2 = \frac{I_\gamma g}{w}$$
Non-collinear Collisions

Calculating Effective Mass Ratio

- Calculate adjusted $\Delta \nu$
  - $\Delta \nu_{edr}$ is the EDR reported $\Delta \nu$
  - $g$ is EMR
  - $\Delta \nu_{adjusted}$ is the EMR Adjusted $\Delta \nu$

$$\Delta \nu_{adjusted} = \frac{\Delta \nu_{edr}}{\gamma}$$
Non-collinear Collisions

Calculating Effective Mass Ratio

- Andy Rich Table and graph
  - Closely approximate your vehicle on the graph
  - Utilize the table to estimate your EMR.
Non-collinear EDR
Case Study #3

✓ Volkswagen
✓ 2011 Lexus ES350
✓ Momentum Analysis
✓ Lexus ES350 covered by CDR Tool
  ✓ Seatbelt
  ✓ Ignition Cycles at Deployment
  ✓ Ignition Cycles at Investigation
  ✓ LATERAL $\Delta \nu$
  ✓ LONITUDINAL $\Delta \nu$
✓ Precrash information
  ✓ Approx 5 sec
  ✓ Typical Speed, brake, etc.
Non-collinear EDR
Case Study #3

- Volkswagen
- 2011 Lexus ES350
- Momentum Analysis
- Lexus ES350 covered by

<table>
<thead>
<tr>
<th>CDR File Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>User Entered VIN/Frame Number: JTHBK1EG1B2419138</td>
</tr>
<tr>
<td>User: D. Maggio</td>
</tr>
<tr>
<td>Case Number: 15-23849</td>
</tr>
<tr>
<td>EDR Data Imaging Date: 03/06/2017</td>
</tr>
<tr>
<td>Crash Date: 11/01/2015</td>
</tr>
<tr>
<td>Filename: JTHBK1EG1B2419138 _ACM.CDRX</td>
</tr>
<tr>
<td>Saved on: Monday, March 6 2017 at 03:19:55</td>
</tr>
<tr>
<td>Imaged with CDR version: Crash Data Retrieval Tool 17.2</td>
</tr>
<tr>
<td>Imaged with Software Licensed to (Company Name): Maricopa County Sheriff</td>
</tr>
<tr>
<td>Reported with CDR version: Crash Data Retrieval Tool 17.6.1</td>
</tr>
<tr>
<td>Reported with Software Licensed to (Company Name): Scottsdale Police Department</td>
</tr>
<tr>
<td>EDR Device Type: Airbag Control Module</td>
</tr>
<tr>
<td>Event(s) recovered: Front/Rear (1), Side (1)</td>
</tr>
</tbody>
</table>
Non-collinear EDR
Case Study #3

- Volkswagen
- 2011 Lexus ES350
- Momentum Analysis
- Lexus ES350 covered by

<table>
<thead>
<tr>
<th>Data Element Name</th>
<th>Positive Sign Notation Indicates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. Longitudinal DeltaV</td>
<td>Forward</td>
</tr>
<tr>
<td>Longitudinal DeltaV</td>
<td>Forward</td>
</tr>
<tr>
<td>Max. Lateral DeltaV, B-Pillar Sensor</td>
<td>Outside to Inside</td>
</tr>
<tr>
<td>Max. Lateral DeltaV, C-Pillar Sensor</td>
<td>Outside to Inside</td>
</tr>
<tr>
<td>Max. Lateral DeltaV, Front Door Sensor</td>
<td>Outside to Inside</td>
</tr>
<tr>
<td>Max. Lateral DeltaV, Slide Door Sensor</td>
<td>Outside to Inside</td>
</tr>
<tr>
<td>Lateral DeltaV, B-Pillar Sensor</td>
<td>Outside to Inside</td>
</tr>
<tr>
<td>Lateral DeltaV, C-Pillar Sensor</td>
<td>Outside to Inside</td>
</tr>
<tr>
<td>Lateral DeltaV, Airbag ECU Sensor</td>
<td>Left to Right</td>
</tr>
<tr>
<td>Roll Angle Peak</td>
<td>Clockwise Rotation</td>
</tr>
<tr>
<td>Roll Angle</td>
<td>Clockwise Rotation</td>
</tr>
<tr>
<td>Lateral Acceleration, Airbag ECU Sensor *</td>
<td>Right to Left</td>
</tr>
</tbody>
</table>

* For sensing a rollover
Non-collinear EDR
Case Study #3

Momentum Analysis

**Volkswagen**
- \( w_1 = 3440 \text{ lbs} \)
- \( \alpha = 0 \)
- \( \theta = 32 \)
- \( v_3 = 41.93 \text{ mph} \)
- \( v_1 = 78.31 \text{ mph} \)

**Lexus ES350**
- \( w_2 = 3695 \text{ lbs} \)
- \( \psi = 90 \)
- \( \phi = 22 \)
- \( v_4 = 42.94 \text{ mph} \)
- \( v_2 = 36.77 \text{ mph} \)

Pre-Crash Data, -5 to 0 seconds (Mo)

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>-4.7</th>
<th>-3.7</th>
<th>-2.7</th>
<th>-1.7</th>
<th>-0.7</th>
<th>0 (TRG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle Speed (MPH [km/h])</td>
<td>36 [58]</td>
<td>37.3 [60]</td>
<td>37.3 [60]</td>
<td>37.3 [60]</td>
<td>37.3 [60]</td>
<td>37.3 [60]</td>
</tr>
<tr>
<td>Brake Switch</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
</tr>
<tr>
<td>Accelerator Rate (V)</td>
<td>1.21</td>
<td>1.25</td>
<td>1.21</td>
<td>1.13</td>
<td>1.17</td>
<td>1.25</td>
</tr>
<tr>
<td>Engine RPM (RPM)</td>
<td>1,200</td>
<td>1,200</td>
<td>1,200</td>
<td>1,200</td>
<td>1,200</td>
<td>1,200</td>
</tr>
</tbody>
</table>
# Non-collinear EDR

## Case Study #3

### Momentum Analysis

<table>
<thead>
<tr>
<th></th>
<th>Volkswagen</th>
<th>Lexus ES350</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>3440 lbs</td>
<td>3695 lbs</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0</td>
<td>$\psi$ = 90</td>
</tr>
<tr>
<td>$\theta$</td>
<td>32</td>
<td>$\phi$ = 22</td>
</tr>
<tr>
<td>$v_3$</td>
<td>41.93 mph</td>
<td>$v_4$ = 42.94 mph</td>
</tr>
<tr>
<td>$v_1$</td>
<td>78.31 mph</td>
<td>$v_2$ = 36.77 mph</td>
</tr>
</tbody>
</table>

**Lexus ES350 Calculated Lateral $\Delta v$**

\[ \Delta v_y = v_4 \sin(\beta) \]

$\beta = \psi - \phi$

$\beta = 90^\circ - 22^\circ$

$\beta = 68^\circ$

\[ \Delta v_y = (42.94)(\sin(68^\circ)) \]

$\Delta v_y = 39.8 \text{ mph}$
Non-collinear EDR
Case Study #3

✓ EDR Data \( \Delta v \) Data
  ✓ Longitudinal \( \Delta v \)
  ✓ Lateral \( \Delta v \)
✓ Which is important to us in this case?
✓ Did the entire crash pulse get recorded?
✓ What is the \( \Delta v \) we should use?
Non-collinear EDR
Case Study #3

✓ Lateral $\Delta v$ Data
✓ Did the entire crash pulse get recorded?
✓ What is the $\Delta v$ we should use?

<table>
<thead>
<tr>
<th>t(ms)</th>
<th>Lat $\Delta v$ (mph)</th>
<th>Lat $\Delta v$ (fps)</th>
<th>g's</th>
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<tbody>
<tr>
<td>-23</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>-19</td>
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<td>-15</td>
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</tr>
<tr>
<td>-11</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>-7</td>
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<tr>
<td>-3</td>
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<td>-0.15</td>
<td>-1.14</td>
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<tr>
<td>1</td>
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<td>0.15</td>
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</tr>
<tr>
<td>5</td>
<td>0.7</td>
<td>1.03</td>
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</tr>
<tr>
<td>9</td>
<td>2</td>
<td>2.93</td>
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<td>13</td>
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<td>17</td>
<td>5</td>
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<tr>
<td>21</td>
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<td>17.07</td>
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<td>25</td>
<td>8.2</td>
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<td>29</td>
<td>9.9</td>
<td>14.51</td>
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<td>33</td>
<td>11.7</td>
<td>17.15</td>
<td>20.49</td>
</tr>
<tr>
<td>37</td>
<td>13.4</td>
<td>19.64</td>
<td>19.35</td>
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<td>41</td>
<td>15.1</td>
<td>22.14</td>
<td>19.35</td>
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<tr>
<td>45</td>
<td>16.5</td>
<td>24.19</td>
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<tr>
<td>49</td>
<td>17.4</td>
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<td>10.24</td>
</tr>
<tr>
<td>53</td>
<td>18.6</td>
<td>27.27</td>
<td>13.66</td>
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<tr>
<td>57</td>
<td>19.7</td>
<td>28.88</td>
<td>12.52</td>
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<td>61</td>
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<td>30.05</td>
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<td>65</td>
<td>20.9</td>
<td>30.64</td>
<td>4.55</td>
</tr>
<tr>
<td>69</td>
<td>21.1</td>
<td>30.93</td>
<td>2.28</td>
</tr>
</tbody>
</table>

$\Delta v_{edr} = 21.1 \text{mph}$

$21.1 \text{mph} \neq 39.8 \text{mph}$
Non-collinear EDR
Case Study #3

✓ Lateral $\Delta v$ Data
✓ Adjust for Effective Mass Ratio
  ✓ $I_y$ found on Autostats
  ✓ Used to calculate $k^2$
  ✓ Need to know $h$

$$k^2 = \frac{I_y g}{w} \quad \gamma = \frac{k^2}{k^2 + h^2}$$

$\Delta v_{edr} = 21.1 \text{mph}$

$h = 4.7 \text{ ft}$

$\gamma =$
Non-collinear EDR
Case Study #3

✓ Lateral $\Delta v$ Data
✓ Adjust for Effective Mass Ratio
  ✓ $I_y$ found on Autostats
  ✓ Used to calculate $k^2$
  ✓ Need to know $h$

$k^2 = \frac{I_y g}{w}$

$k^2 = \frac{(2481.1 \text{ lb}\cdot \text{ft}\cdot \text{sec}^2)(32.2 \text{ ft} / \text{sec}^2)}{3695 \text{ lb}}$

$k^2 = \frac{(78,891.4 \text{ lb}\cdot \text{ft}^2)}{3695 \text{ lb}}$

$k^2 = 21.35 \text{ ft}^2$

$\gamma = \frac{k^2}{k^2 + h^2}$

$\gamma = \frac{21.35 \text{ ft}^2}{21.35 \text{ ft}^2 + (4.7 \text{ ft})^2}$

$\gamma = \frac{21.35 \text{ ft}^2}{21.35 \text{ ft}^2 + 22.09 \text{ ft}^2}$

$\gamma = \frac{21.35 \text{ ft}^2}{43.44 \text{ ft}^2}$

$\gamma = 0.4914$

$\Delta v_{edr} = 21.1 \text{ mph}$

$k^2 = 21.35 \text{ ft}^2$

$h = 4.7 \text{ ft}$

$\gamma = 0.4914$
Non-collinear EDR
Case Study #3

Effective Mass Ratio for Various Vehicle Types

- Honda Fit
- Toyota Camry
- Crown Vic
- Honda CRV
- Chev Tahoe
- F150 Xtra Cab Pickup

\[ \Delta v_{edr} = 21.1 \, \text{mph} \]
\[ k^2 = 21.35 \, \text{ft}^2 \]
\[ h = 4.7 \, \text{ft} \]
\[ \gamma = 0.4914 \]

Authored by Andrew Rich
Non-collinear EDR
Case Study #3

<table>
<thead>
<tr>
<th>h (ft)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>h (m)</td>
<td>0</td>
<td>0.30</td>
<td>0.61</td>
<td>0.91</td>
<td>1.22</td>
<td>1.52</td>
<td>1.83</td>
<td>2.13</td>
<td>2.44</td>
</tr>
<tr>
<td>EMR</td>
<td>1</td>
<td>0.95</td>
<td>0.83</td>
<td>0.7</td>
<td>0.57</td>
<td>0.45</td>
<td>0.37</td>
<td>0.3</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Soooooo....

Comparing each method you can see, it doesn’t make much difference which way you do it!

Mathemagically: 0.4914
Graphically: 0.49
Table: 0.486

$\Delta v_{edr} = 21.1 \text{ mph}$
$k^2 = 21.35 \text{ ft}^2$
$h = 4.7 \text{ ft}$
$\gamma = 0.4914$
Non-collinear EDR
Case Study #3

- Lateral $\Delta v$ Data
- Adjust for Effective Mass Ratio

$$
\Delta v_{\text{adjusted}} = \frac{\Delta v_{\text{edr}}}{\gamma}
$$

$$
\Delta v_{\text{adjusted}} = \frac{21.1 \text{mph}}{0.4914}
$$

$$
\Delta v_{\text{adjusted}} = 42.93 \text{mph}
$$

You must also consider edr error:

Low

$$
\Delta v_{\text{adjusted}} = \frac{\Delta v_{\text{edr}}}{\gamma}
$$

$$
\Delta v_{\text{adjusted}} = \frac{18.99 \text{mph}}{0.4914}
$$

$$
\Delta v_{\text{adjusted}} = 38.64 \text{mph}
$$

Calculated 39.8 mph

Target

$$
\Delta v_{\text{adjusted}} = \frac{\Delta v_{\text{edr}}}{\gamma}
$$

$$
\Delta v_{\text{adjusted}} = \frac{21.1 \text{mph}}{0.4914}
$$

$$
\Delta v_{\text{adjusted}} = 42.93 \text{mph}
$$

High

$$
\Delta v_{\text{adjusted}} = \frac{\Delta v_{\text{edr}}}{\gamma}
$$

$$
\Delta v_{\text{adjusted}} = \frac{23.21 \text{mph}}{0.4914}
$$

$$
\Delta v_{\text{adjusted}} = 47.23 \text{mph}
$$
Non-collinear EDR
Case Study #3

✓ Lateral $\Delta v$ Data
✓ Remember:

$$\Delta V_1 = -\Delta V_2 \frac{W_2}{W_1}$$

$$\Delta v_{1x} = -\Delta v_{2y} \left( \frac{W_2}{W_1} \right)$$

How can the vectors be made co-linear?
Non-collinear EDR
Case Study #3

✓ Lateral $\Delta v$ Data
✓ Remember:

$\Delta v_{2y} = 39.8 \text{mph}$
$\Delta v_{1x} = \frac{W_2}{W_1}$
Non-collinear EDR
Case Study #3

Momentum Analysis

<table>
<thead>
<tr>
<th>Volkswagen</th>
<th>Lexus ES350</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1=3440$ lbs</td>
<td>$w_2=3695$ lbs</td>
</tr>
<tr>
<td>$\alpha = 0$</td>
<td>$\psi = 90$</td>
</tr>
<tr>
<td>$\theta = 32$</td>
<td>$\phi = 22$</td>
</tr>
<tr>
<td>$v_3 = 41.93$ mph</td>
<td>$v_4 = 42.94$ mph</td>
</tr>
<tr>
<td>$v_1 = 78.31$ mph</td>
<td>$v_2 = 36.77$ mph</td>
</tr>
</tbody>
</table>

Volkswagen Calculated Longitudinal Post Impact Velocity:

\[
v_{3x} = v_4 s \cos(\theta) = 41.93 \cos(32^\circ) = 35.55 \text{ mph}
\]

\[
\Delta v_{1x} = -\Delta v_{2y} \left( \frac{w_2}{w_1} \right)
\]

\[
\Delta v_{2y} = 39.8 \text{ mph}
\]

\[
\Delta v_{1x} = \quad v_{3x} = 35.55 \text{ mph}
\]
Non-collinear EDR
Case Study #3

✓ Lateral $\Delta v$ Data
✓ Remember:

$V_{3x} = 35.55\text{mph}$

$\Delta v_{1x} = \Delta v_{2y} \frac{w_2}{w_1}$

$\Delta v_{2y} = 39.8\text{mph}$
$\Delta v_{1x} = v_{3x} = 35.55\text{mph}$
Non-collinear EDR

Case Study #3

✓ Lateral $\Delta v$ Data
✓ Remember:

$\Delta v_{2y} = 39.8\text{mph}$

$\Delta v_{1x} = \frac{w_2}{w_1}$
Non-collinear EDR
Case Study #3

✓ Lateral $\Delta v$ Data
✓ Remember:

$$\Delta v_{1x} = -\Delta v_{2y} \left( \frac{w_2}{w_1} \right)$$

$$\Delta v_{1x} = -(39.8) \left( \frac{3695}{3440} \right)$$

$$\Delta v_{1x} = -39.8(1.07)$$

$$\Delta v_{1x} = -42.58$$

$$\Delta v_{2y} = 39.8 \text{ mph}$$
$$\Delta v_{1x} = -42.58 \text{ mph}$$
$$v_{3x} = 35.55 \text{ mph}$$
Non-collinear EDR
Case Study #3

✓ Lateral $\Delta v$ Data
✓ Remember:

$v_1 = v_{3x} + \Delta v_{1x}$

$v_1 = 35.55 + 42.58$

$v_1 = 78.13$ mph

$v_{3x} = 35.55$ mph

$\Delta v_{1x} = -42.58$ mph

$\Delta v_{2y} = 39.8$ mph

$v_{3x} = 35.55$ mph

$\Delta v_{1x} = -\Delta v_{2y} \left( \frac{w_2}{w_1} \right)$
How about triangles?

Let’s look at the info we have from the EDR in the Lexus

• What do we have?
  – Impact Velocity, \((V_2)\), EDR Report
  – Post Impact Velocity, \((V_3)\), Calculated
  – Angle \(\beta\), \((\psi \square \phi)\)

• With three sides, we can get an opposite side of one of the angles, in this case, law of cosines.

\[
\Delta v = \sqrt{v_2^2 + v_4^2 - 2v_2v_4 \cos \beta}
\]
How about triangles?

Let’s look at the info we have from the EDR in the Lexus

- What do we have?
  - Impact Velocity, \( V_2 \), EDR Report
  - Post Impact Velocity, \( V_4 \), Calculated
  - Angle \( \beta \), \((\psi \varphi)\)
- With an opposite pair we can now use the law of sines:

\[
\frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{C}
\]
How about triangles?

Let’s look at the info we have from the EDR in the Lexus

• What do we have?
  – Impact Velocity, \( (V_2) \), EDR Report
  – Post Impact Velocity, \( (V_3) \), Calculated
  – Angle \( \beta \), \( (\psi \ □ \ \phi) \)

• With an opposite pair we can now use the law of sines:

\[
\frac{\sin \alpha}{v_4} = \frac{\sin \beta}{\Delta v_2} = \frac{\sin RA}{v_2}
\]
How about triangles?

Let’s look at the info we have from the EDR in the Lexus

- What can we get for the Volkswagen?
  - Use $v_3$ from reconstruction
  - $\alpha_1$ is opposite, so the angle must be the complimentary angle to $\alpha_2$ ($\alpha_2 - 90^\circ = \alpha_1$)
  - $\Delta v_1$ can be calculated with:

$$\Delta v_1 = -\Delta v_2 \left( \frac{w_2}{w_1} \right)$$
How about triangles?

Let’s look at the info we have from the EDR in the Lexus

- Use law of sines:

\[
\frac{\sin \alpha}{v_3} = \frac{\sin \theta}{\Delta v_1} = \frac{\sin RA}{v_1}
\]
Non-collinear EDR
Case Study #3

Now let’s look at our crash:

• What do we have?
  – Impact Velocity, \((V_2)\), EDR Report
  – Post Impact Velocity, \((V_3)\), Calculated
  – Angle \(\beta\), \((\psi \square \phi)\)

• With three sides, we can get an opposite side of one of the angles, in this case, law of cosines.
Non-collinear EDR
Case Study #3

Now let’s look at our crash:

• What do we have?
  - $V_2 = 37.3$
  - $V_3 = 42.94$
  - $\beta = 68^\circ$, $(\psi \square \phi)$

\[ \Delta v = \sqrt{v_2^2 + v_4^2 - 2v_2v_4 \cos \beta} \]
\[ \Delta v = \sqrt{(37.3)^2 + (42.94)^2 - 2(37.3)(42.94)(\cos(68))} \]
\[ \Delta v = \sqrt{2035.13} \]
\[ \Delta v = 45.11 \text{mph} \]
Non-collinear EDR
Case Study #3

Now let’s look at our crash:

• What do we have?
  – $V_2 = 37.3$
  – $V_3 = 42.94$
  – $\beta = 68^\circ$, $(\psi \square \phi)$

• With an opposite pair we can now use the law of sines:

\[
\frac{\sin \alpha}{v_4} = \frac{\sin \beta}{\Delta v_2} = \frac{\sin RA}{v_2}
\]

\[
\frac{\sin 62^\circ}{42.94} = \frac{\sin 68^\circ}{45.11} = \frac{\sin 50^\circ}{37.3}
\]
Non-collinear EDR • Now address the Volkswagen triangle

Case Study #3

Now let’s look at our crash:

\[ \frac{\sin 320^\circ}{\Delta \gamma_1} \cdot \frac{\sin 28^\circ}{42.94} = \frac{32^\circ}{120^\circ} \]
Non-collinear EDR
Case Study #3

Summary

<table>
<thead>
<tr>
<th>Momentum Analysis</th>
<th>$\Delta v$ Derived</th>
<th>Triangle Derived</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min: 70 MPH</td>
<td>Min: 76 MPH</td>
<td>Min: 75 MPH</td>
</tr>
<tr>
<td>Target: 78.31 MPH</td>
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Crosschecks:
- Momentum analysis agrees with EDR reported precrash speed of the Lexus
### Non-collinear EDR
### Case Study #3

#### Momentum Analysis

**Volkswagen**
- \( w_1 = 3440 \text{ lbs} \)
- \( \alpha = 0 \)
- \( \theta = 32 \)
- \( v_3 = 41.93 \text{ mph} \)
- \( v_1 = 78.31 \text{ mph} \)

**Lexus ES350**
- \( w_2 = 3695 \text{ lbs} \)
- \( \psi = 90 \)
- \( \phi = 22 \)
- \( v_4 = 42.94 \text{ mph} \)
- \( v_2 = 36.77 \text{ mph} \)

---

**Pre-Crash Data, -5 to 0 seconds (Ms)**

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>-4.7</th>
<th>-3.7</th>
<th>-2.7</th>
<th>-1.7</th>
<th>-0.7</th>
<th>0 (TRG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle Speed (MPH [km/h])</td>
<td>36 [58]</td>
<td>37.3 [60]</td>
<td>37.3 [60]</td>
<td>37.3 [60]</td>
<td>37.3 [60]</td>
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<tr>
<td>Brake Switch</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
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</tr>
<tr>
<td>Accelerator Rate (V)</td>
<td>1.21</td>
<td>1.25</td>
<td>1.21</td>
<td>1.13</td>
<td>1.17</td>
<td>1.25</td>
</tr>
<tr>
<td>Engine RPM (RPM)</td>
<td>1,200</td>
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Non-collinear EDR
Case Study #3
Summary

Crosschecks:

- Momentum analysis agrees with EDR reported precrash speed of the Lexus
- EDR reported Lateral $\Delta v$ agrees with momentum calculated $\Delta v$ when adjusted for EMR

### Momentum Analysis

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Non-collinear EDR
Case Study #3

✓ Lateral $\Delta v$ Data
✓ Adjust for Effective Mass Ratio

\[
\Delta v_{\text{adjusted}} = \frac{\Delta v_{\text{edr}}}{\gamma}
\]

\[
\Delta v_{\text{adjusted}} = \frac{21.1 \text{ mph}}{0.4914}
\]

\[
\Delta v_{\text{adjusted}} = 42.93 \text{ mph}
\]

You must also consider edr error:

Low

\[
\Delta v_{\text{adjusted}} = \frac{\Delta v_{\text{edr}}}{\gamma}
\]

\[
\Delta v_{\text{adjusted}} = \frac{18.99 \text{ mph}}{0.4914}
\]

\[
\Delta v_{\text{adjusted}} = 38.64 \text{ mph}
\]

Calculated 39.8mph

Target

\[
\Delta v_{\text{adjusted}} = \frac{\Delta v_{\text{edr}}}{\gamma}
\]

\[
\Delta v_{\text{adjusted}} = \frac{21.1 \text{ mph}}{0.4914}
\]

\[
\Delta v_{\text{adjusted}} = 42.93 \text{ mph}
\]

High

\[
\Delta v_{\text{adjusted}} = \frac{\Delta v_{\text{edr}}}{\gamma}
\]

\[
\Delta v_{\text{adjusted}} = \frac{23.21 \text{ mph}}{0.4914}
\]

\[
\Delta v_{\text{adjusted}} = 47.23 \text{ mph}
\]
Non-collinear EDR
Case Study #3

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Momentum

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Dv Derived

Triangle Derived